Name: Feedback

SA402 · Dynamic and Stochastic Models | Fall 2022 · Uhan

Quiz 7 – 11/4/2022

Instructions. You have 20 minutes to complete this quiz. You may use your plebe-issue calculator. You may use your own course materials (e.g., notes, homework, website). No collaboration allowed.

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

Problem	Weight	Score
1	1	
2	1	
3	1	
4	1	
Total		/ 40

For Problems 1-3, consider the Markov chain defined by the following one-step transition matrix:

$$\mathbf{P} = \begin{bmatrix} 0.1 & 0.2 & 0.4 & 0.1 & 0.2 \\ 0 & 0.6 & 0.4 & 0 & 0 \\ 0 & 0.9 & 0.1 & 0 & 0 \\ 0.5 & 0.2 & 0.2 & 0 & 0.1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

There are two irreducible sets of states: $\{2, 3\}$ and $\{5\}$.

Problem 1. Is state 3 transient or recurrent? Briefly explain.

Recall from Lesson 9 (top of page 3) that all states in an irreducible set are recurrent, and all states not in an irreducible set are transient.

Problem 2. Suppose the Markov chain has reached state 3. What is the steady-state probability of being in state 2?

Several of you computed the $\pi_{\mathcal{R}}$ vector correctly, but did not specify which component of $\pi_{\mathcal{R}}$ gives the steady-state probability of being in state 2.

See Example 2 in Lesson 9 for a similar example.

Here is the one-step transition matrix from the previous page, for your convenience:

$$\mathbf{P} = \begin{bmatrix} 0.1 & 0.2 & 0.4 & 0.1 & 0.2 \\ 0 & 0.6 & 0.4 & 0 & 0 \\ 0 & 0.9 & 0.1 & 0 & 0 \\ 0.5 & 0.2 & 0.2 & 0 & 0.1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 3. Suppose the Markov chain starts in state 4. What is the probability that the Markov chain is absorbed into state 5?

Note that in the absorption probability formula $\alpha_{\mathcal{TR}} = (\mathbf{I} - \mathbf{P}_{\mathcal{TT}})^{-1}\mathbf{P}_{\mathcal{TR}}$, the set $\mathcal{T}$ consists of all transient states.

Unfortunately, Example 3 in Lesson 9 is somewhat miselading because the example only has 1 transient state. See Problem 1d from the Lesson 9 Exercises for a better example.

Problem 4. Consider a model of an elevator's movement from floor to floor in a high-rise building, in which the state of the system is defined as the floor on which the elevator is currently stopped, and the time index is defined to be the number of stops. Describe what assumptions need to be made in order for the Markov property to hold. (You do not need to discuss whether these assumptions are realistic.)

Many of you described the assumptions needed for both the Markov property and the time stationarity property to hold, but the problem only asked about the Markov property.

See Example 1 in Lesson 10 and Problem 1 in the Lesson 10 Exercises for similar examples.